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Application of the Ewing Equation for Calculating Thermal Conductivity from Electrical Conductivity

A. E. Powers

April 7, 1961

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UC-25, Metals, Ceramics,
and Materials
(TID-4500, 16th Edition)

APPLICATION OF THE EWING EQUATION FOR CALCULATING THERMAL CONDUCTIVITY FROM ELECTRICAL CONDUCTIVITY

A. E. Powers

April 7, 1961

Stuart Sturges
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ABSTRACT

The usefulness of the Ewing equation for calculating the thermal conductivity of reactor metals and alloys from electrical resistance, specific heat, density, and atomic weight has been investigated. The alloys investigated were Zircaloy-2, HSZA, Nb + 5.5 w/o V, Inconel, 18-8 stainless steel, and eutectic NaK. The Ewing equation was found to give calculated values with a degree of confidence similar to that of actual measured values.

APPLICATION OF THE EWING EQUATION FOR CALCULATING THERMAL CONDUCTIVITY FROM ELECTRICAL CONDUCTIVITY

A. E. Powers

INTRODUCTION

Knowledge of accurate values for thermal conductivity of materials is extremely important in the engineering design of nuclear reactors. Fortunately, methods for calculating the over-all conductivity of dispersion materials are well developed mathematically[1,2], but the calculated conductivity thus derived is accurate only to the extent that the conductivity values for the individual component materials are precisely determined.

The experimental determination of thermal conductivity is a difficult and often an unsatisfactory undertaking, as is evidenced by the scarcity of such data and the variance in data originating from different apparatus and laboratories.

For many years theoretical and experimental considerations have shown a correspondence between thermal and electrical conductivity. This correspondence was one of the prime considerations during the pioneer developments in the modern theory of metals. Wiedemann and Franz first pointed out, in 1853, that the ratio of the two conductivities, K/σ, is fairly constant for all metals at room temperature[3]. Lorentz, in 1881, then determined that K/σ is proportional to temperature[4], and, in 1900, Drude[5] showed that the Lorentz constant $K/\sigma T = L = 3(k_o/e)^2$, where k_o is the Boltzmann constant and e is the electron charge. A further development of the electron theory of metals by Sommerfeld in 1928 using Fermi-Dirac statistics showed the constant to be $\frac{\pi^2}{3} \left(\frac{k_o}{e}\right)^2 = 2.45 \times 10^{-8}$ when the units are expressed in the cgs system.[6]

Although thermal conductivity in metals is largely electronic, some of the heat is conducted by the atomic lattice.[2] An approximate relation is therefore

$$K = L\, \sigma T + k \tag{1}$$

where the term $L\,\sigma\,T$ is considered the electronic contribution and k is the lattice contribution. The constants L and k vary from metal to metal and with temperature. Therefore, the above relation is not of much use in calculating thermal conductivity if only electrical conductivity is known, since L and k are also unknown. Occasionally, investigators have attempted to devise an equation for thermal conductivity based on the physical properties of the metal and related to Equation 1. The most recent, and perhaps the most successful, is the semi-empirical equation of Ewing, Walker, Grand, and Miller of the U.S. Naval Research Laboratory.[7]

THE EWING EQUATION

Ewing, et al., correlated the thermal and electrical conductivities of 140 metals and alloys, both liquid and solid, developing the equation

$$K = 2.61 \times 10^{-8}\left(\frac{T}{\rho}\right) - 2 \times 10^{-17}\left(\frac{T}{\rho}\right)^2 \frac{1}{C_p d} + 97\,\frac{C_p d^2}{MT} \quad . \qquad (2)$$

where K = thermal conductivity in watts/cm^2/cm/$^\circ$C.

ρ = electrical resistance in ohm-cm

C_p = specific heat in cal/gm, $^\circ$C

d = density in gm/cc

M = average atomic or molecular weight .

Ewing et al. found the mean deviation for liquid metals to be 12% and for solid alloys less than 5%. The equation has also been used for determining the degree of association of organic liquids with a fair degree of success.

The relation is an expansion of $K = L\sigma T + k$ with the Lorentz constant equal to 2.61×10^{-8} rather than the theoretical 2.45×10^{-8}. The second term is a slight correction to the electronic component and the third term is considered to be related to the lattice component. The third term implies that atomic conduction at unit gradient is directly proportional to the energy difference across a centimeter cube, directly proportional to the number of ions per centimeter cube, and inversely proportional to the absolute temperature.

COMPARISON OF CALCULATED TO EXPERIMENTAL VALUES

The metals involved in the Ewing calculation of thermal conductivity were Zircaloy-2, Zr + 2.7 w/o Sn + 2.0 w/o Nb (HSZA), Nb + 5.5 w/o V, Inconel, Type 304 stainless steel, and NaK. The electrical resistivities are given in Figure 1 and the compositions of the Zircaloys, HSZA, and Nb + 5.5 w/o V are given in Table 1. Curves of experimental and calculated values of thermal conductivity are illustrated in Figures 2 through 7. Sources of the data and the bibliographic reference numbers are enclosed in brackets on each figure.

There are two calculated curves in Figure 2 for Zircaloy, both using the same electrical resistance data. One is from the Ewing equation and the other is from an equation for zirconium-base alloys devised by Bing, Fink, and Thompson at the Battelle Memorial Institute.[10] Although the Ewing equation lies about 6% below the mean of the experimental curves, it agrees more closely with the experimental data than does the Bing equation.

TABLE I

COMPOSITION OF ZIRCALOYS, HSZA, AND NIOBIUM + 5.5 VANADIUM

	Zircaloy-2 [BMI8]	Zircaloy-4 [BMI8]	HSZA [BMI8]	Nb + 5.5 w/o V [BMI8]
Sn	1.47	1.32	2.72	
Nb			2.18	
Fe	0.125	0.152	0.071	
Cr	0.086	0.099	0.006	
Ni	0.056	0.001		
V				5.54
Ta				0.052
Si	51 ppm		60 ppm	
Al	38 ppm		34 ppm	
C	65 ppm	150 ppm	80 ppm	65 ppm
N	45 ppm	36 ppm	46 ppm	160 ppm
O	1489 ppm	1380 ppm	1260 ppm	142 ppm
H	11 ppm	30 ppm	6 ppm	
Cu	20 ppm	20 ppm	40 ppm	
Hf	<125 ppm	60 ppm	60 ppm	

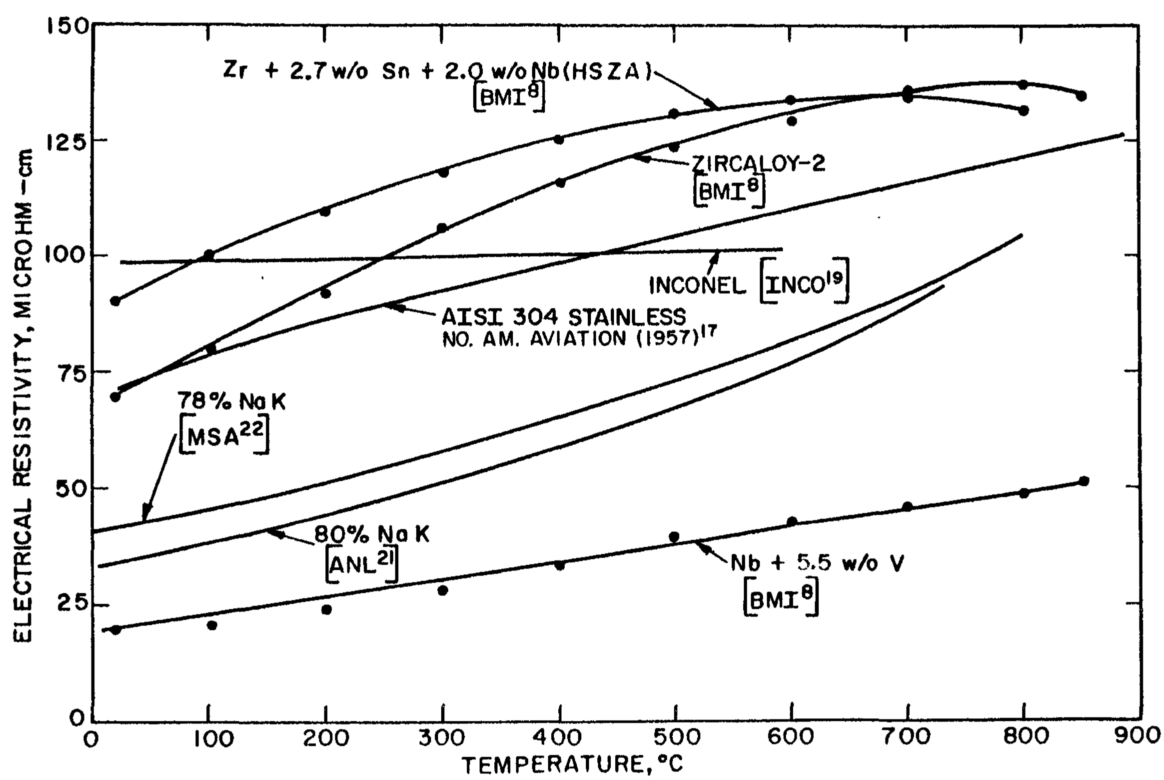

FIGURE 1. Electrical Resistivity of Alloys.
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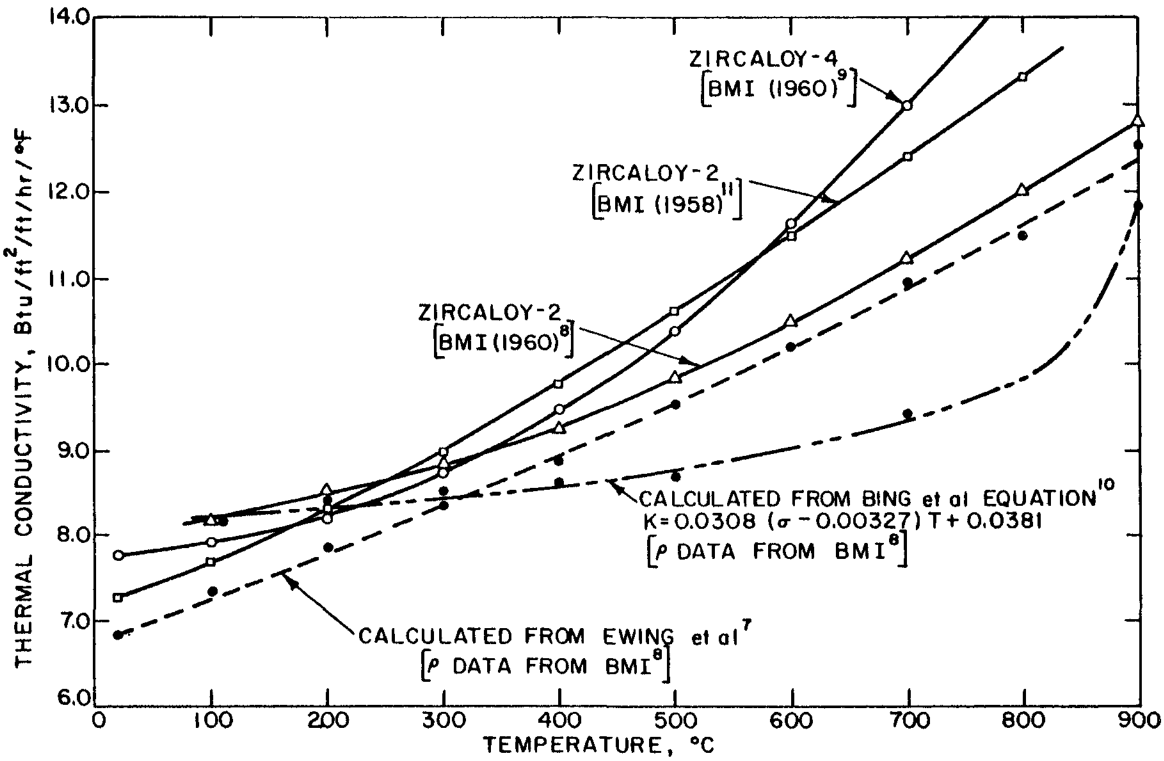

FIGURE 2. Thermal Conductivity of Zircaloy-2 and -4 Determined Experimentally and Calculated from Electrical Resistance.

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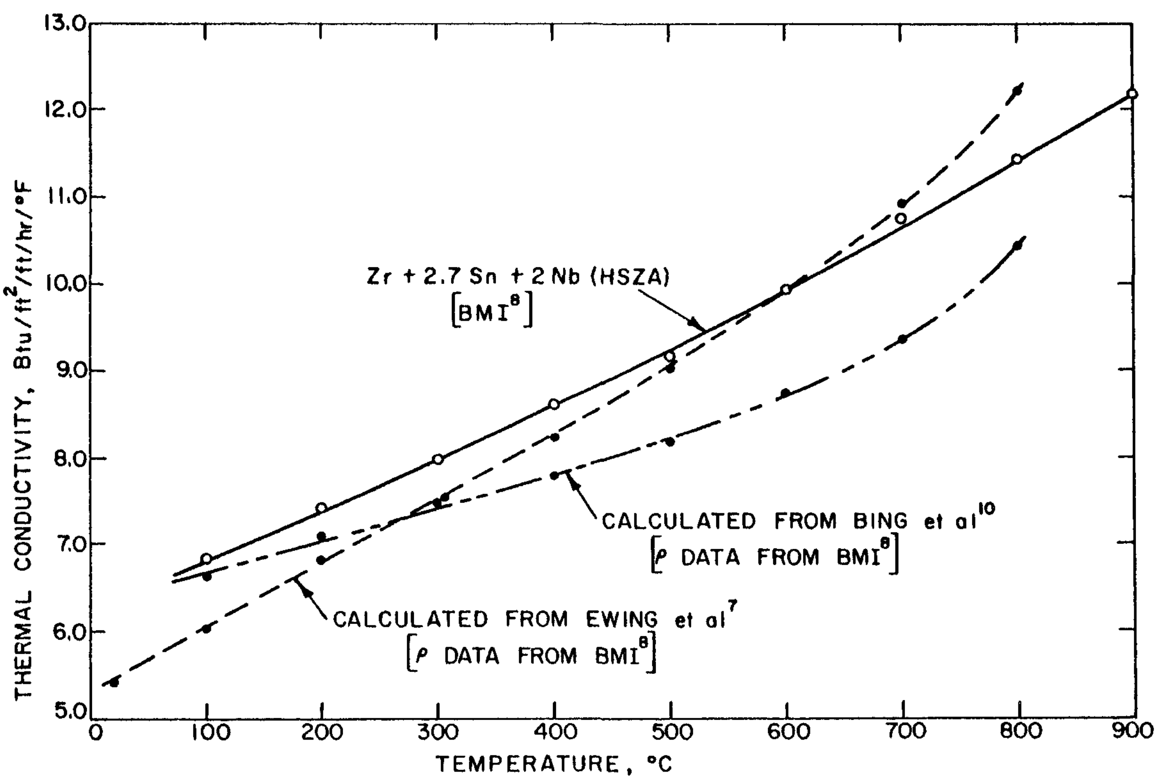

FIGURE 3. Thermal Conductivity of HSZA Determined Experimentally and Calculated from Electrical Resistance.

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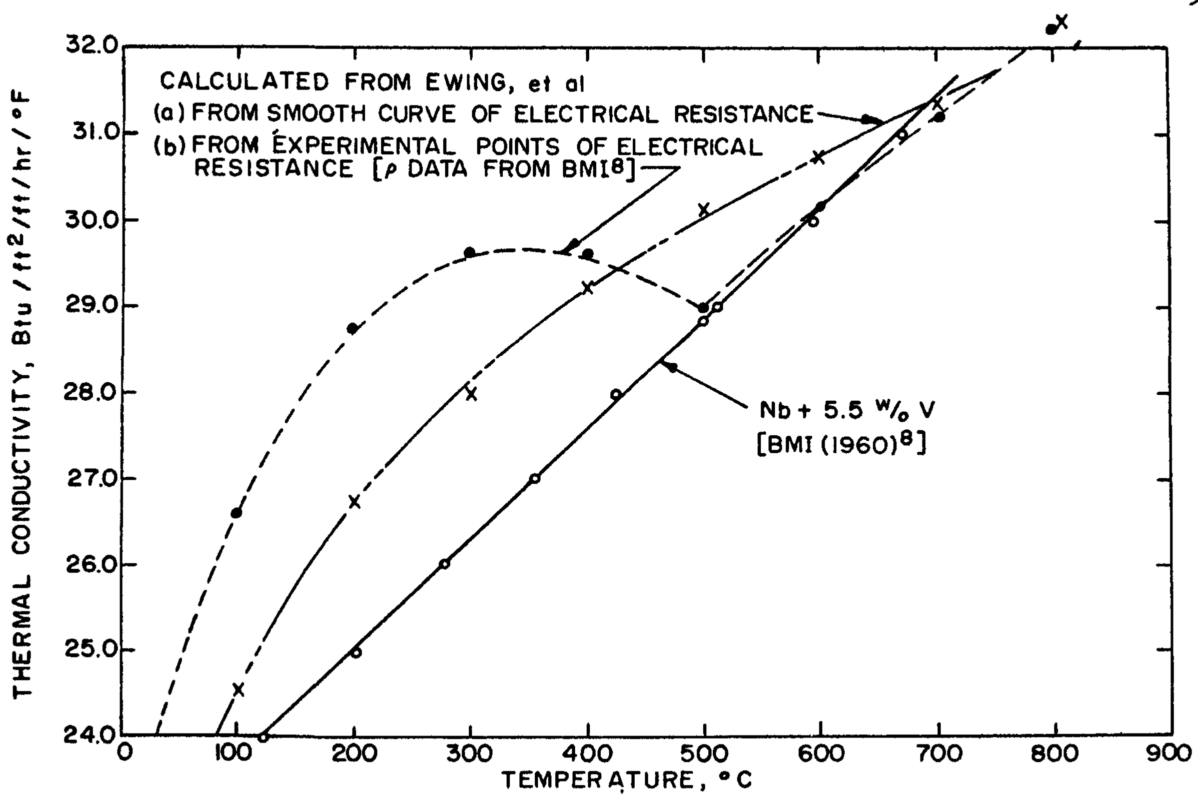

FIGURE 4. Thermal Conductivity of Nb+ 5.5 w/o V Determined Experimentally and Calculated from Electrical Resistance.

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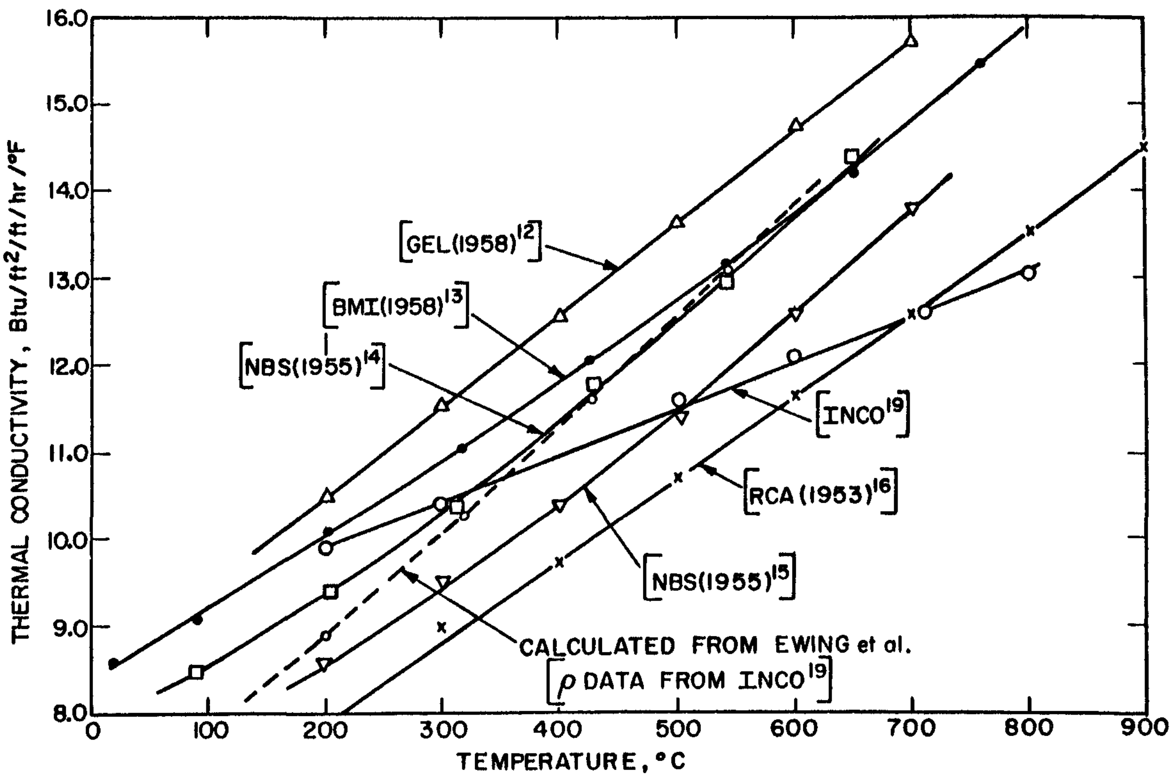

FIGURE 5. Thermal Conductivity of Inconel Determined Experimentally and Calculated from Electrical Resistance.

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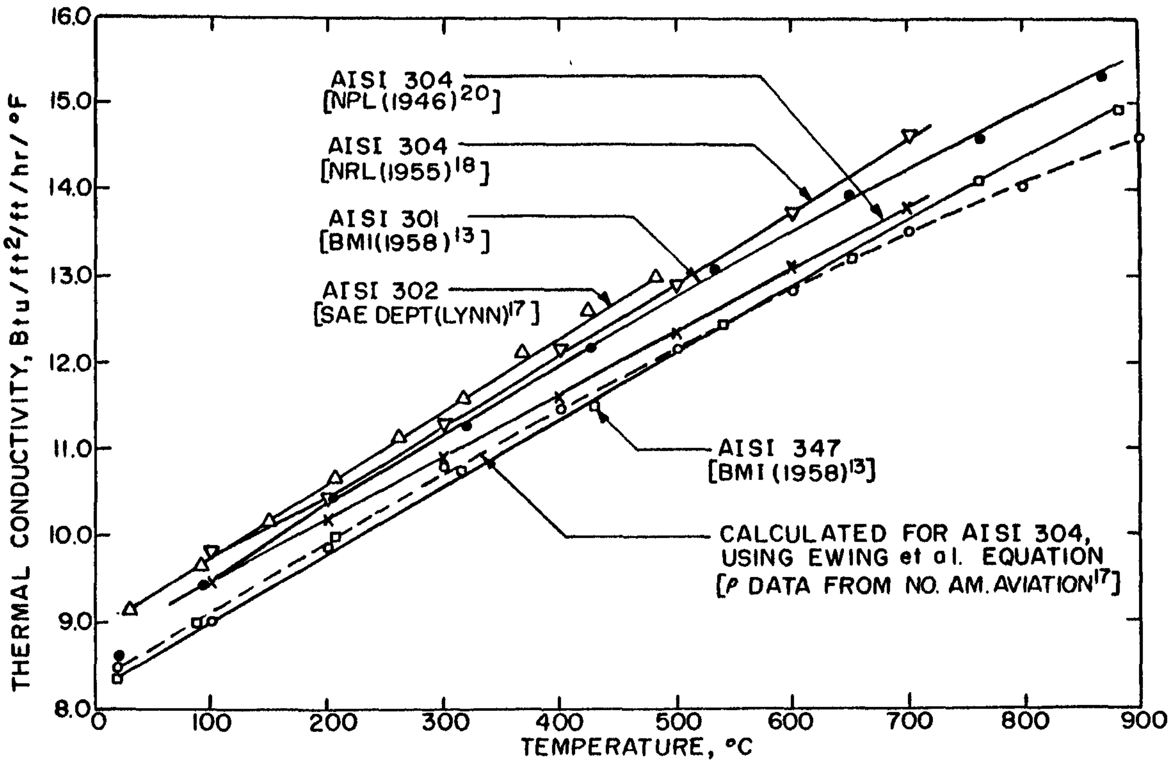

FIGURE 6. Thermal Conductivity of 18-8 Stainless Steel Determined Experimentally and Calculated from Electrical Resistance.

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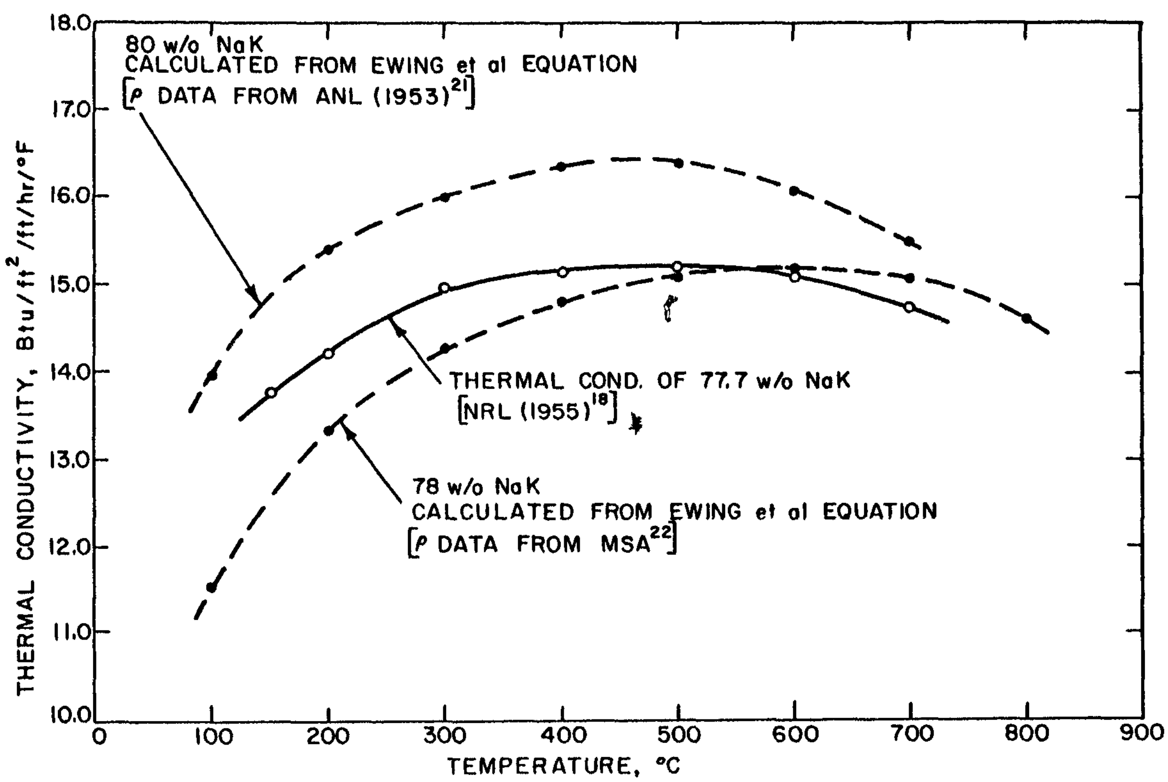

FIGURE 7. Thermal Conductivity of Na - 78 w/o K Determined Experimentally and Calculated from Electrical Resistance.

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Two calculated curves from the Bing and the Ewing equations are presented in Figure 3 for HSZA and again the Ewing equation is in closer agreement with the experimental data.

The data for Nb + 5.5 V in Figure 4 illustrate the difficulty presented by unsatisfactory electrical resistance values. The original data in Figure 1 possess some scatter and when the actual data points are used in the calculation, curve b of unsatisfactory nature results. Curve a was calculated from values taken from the smooth curve of electrical resistance rather than from the original data points. Curve a is still not in satisfactory conformance with the experimental curve.

For most metals and alloys, electrical conductivity data over a wide range of temperatures are more rare than thermal conductivity data. Only one set of data could be found for Zircaloy, HSZA, Nb + 5.5 V, Inconel, and 304 stainless steel. The single calculated curve in Figure 5 for Inconel lies approximately in the middle of a wide array of experimental curves.

The curves for several 18-8 stainless steels in Figure 6 are grouped together more closely than in the case of Inconel, and the single calculated curve lies along the lowest experimental curve.

Figure 7 presents two calculated curves and one experimental curve for a liquid metal -- eutectic Na-K alloy. It is obvious in this instance, as it has been for all the other alloys, that the congruity of calculated to experimental curves must depend, first of all, on the availability of accurate data for both thermal conductivity and electrical conductivity.

It is interesting to note in the calculations the proportion of lattice conductivity to total conductivity as indicated by the Ewing equation. The proportion of lattice conductivity diminishes as temperature increases. In Zircaloy-2, for example, it is 9% of the total at 20°C and 1.8% at 900°C. At room temperature, the ratios are 11% for HSZA, 4% for Nb + 5.5 V, 34% for Inconel, 27% for 304 stainless steel, and about 0.6% for eutectic NaK. These calculations may be inspected by the reader upon inquiry to the author.

DISCUSSION

The Ewing equation appears to be one of the best yet devised for calculating the thermal conductivity of solid and liquid metals and alloys from known values of electrical resistance, specific heat, density, and atomic weight. In most cases, by using the Ewing equation, thermal conductivity can be calculated with about the same degree of confidence as it can be measured directly. However, greater usefulness of this equation is hindered by the lack of accurate data on both electrical and thermal conductivity. With the availability of more accurate data, the Ewing equation could probably be modified to permit more precise calculations. For most metals, electrical-conductivity data over a range of temperature are more rare than comparable thermal-conductivity data.

The major use of the Ewing equation is in determining the approximate thermal conductivity and temperature coefficient for new alloys in a manner more economical than direct measurement. It can be of particular benefit in determining the directional thermal conductivity of dispersion-type materials. The applicability of this equation to nonmetallic materials has not been tested, although it contains a term for lattice conduction. Nevertheless, it might be particularly useful in determining anisotropy of conductivity in dispersion-type material.[1] But before engineering values of thermal conductivity in dispersion materials can be obtained from electrical resistance measurements, accurate determinations of both thermal and electrical conductivity must be available for the dispersion materials. The obtainment of such data, for the purpose of developing a Wiedemann-Franz type equation for application to metallic-nonmetallic dispersion systems, should be considered of high importance to the development of nuclear fuel materials.

CONCLUSIONS

The application of the Ewing equation should prove valuable in determining thermal conductivity from electrical resistance for reactor materials, particularly for composite and dispersion materials. There is an important need for accurate measurements of thermal and electrical conductivity on dispersion materials so that the Ewing-type equation may be developed for greater precision.

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